Imperative programming with Python

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Functions: DIY

- Functions are defined with the `def` keyword.

```
def is_even(n):
    if n % 2 == 0:
        return True
    else:
        return False
```

- The argument passed to `is_even(n)` will be assigned to `n`.
- The `return` keyword sets the return value and exits the function immediately. It can also be used without a value (just `return`).
- *Good practice tip*: reduce the number of return points. If possible, have only one.

```
def is_even(n):
    return (n % 2 == 0)
```

Functions: local variables

- Variables inside function definitions have a *local* scope.

```
def average(n, m):
    thesum = float(n + m)
    return thesum/2
```

- You can only use the function as a *black box*

```
>>> print average(3,4)
3.5
>>> print thesum
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'thesum' is not defined
```

- Design tip: thinking of functions as black boxes performing a certain action is the way to go.

Functions: execution and the call stack

```
def cat_twice_and_print(part1, part2):
    cat = part1 + part2
    print_twice(cat)

def print_twice(msg):
    print msg
    print msg

line1 = 'welcome␣'
line2 = 'to␣the␣jungle'
cat_twice_and_print(line1, line2)
```

print_twice
msg ↦ 'welcome to the jungle'

cat_twice_and_print
part1 ↦ 'welcome '
part2 ↦ 'to the jungle'
cat ↦ 'welcome to the jungle'

__main__
line1 ↦ 'welcome '
line2 ↦ 'to the jungle'

Output:

```
welcome to the jungle
welcome to the jungle
```

Functions: recursion

- Functions can call *themselves* in their definition.

```
# calculates n * m (in a complicated way)
def multiply(n, m):
    if n == 0:
        return 0
    else:
        return m + multiply(n - 1, m)
```

- How does the call stack look for `multiply(2, 7)` look like?

multiply $\texttt{n} \mapsto 2$, $\texttt{m} \mapsto 7$ $\texttt{ret} \mapsto 7 + \ldots$
multiply $\texttt{n} \mapsto 1$, $\texttt{m} \mapsto 7$ $\texttt{ret} \mapsto 7 + \ldots$
multiply $\texttt{n} \mapsto 0$, $\texttt{m} \mapsto 7$ $\texttt{ret} \mapsto 0$

Functions: recursion

- It is crucial that the arguments of a recursive call are in some sense 'smaller' than the arguments of the function call itself.
- What happens if we write `multiply` as follows

```
def multiply(n, m):
    if n == 0:
        return 0
    else:
        return m + multiply(n, m)
```

```
>>> multiply(2, 7)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "<stdin>", line 5, in multiply
  File "<stdin>", line 5, in multiply
...
  File "<stdin>", line 5, in multiply
RuntimeError: maximum recursion depth exceeded
```

- Stack overflow!

Functions: recursion

- You can also have many recursive calls

```
def fib(n):
    if n == 0:
        return n
    else:
        return fib(n - 1) + fib(n - 2)
```

- Is it well defined? No, what about `fib(1)` ?

```
def fib(n):
    if n == 0 or n == 1:
        return n
    else:
        return fib(n - 1) + fib(n - 2)
```

- Is it well defined? Yes.

Functions: recursion

- The argument itself *can* increase...

```
def reverse_string(s):
    return reverse_from_n(s, 0)

def reverse_from_n(s, i):
    if i == len(s):
        return ''
    else:
        return reverse_from_n(s, i+1) + s[i]
```

- But if you look closer `len(s) - i` is strictly decreasing.

Functions: recursion

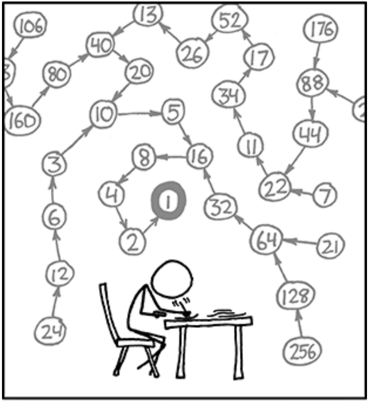
- Is the folowing function well defined (for $n > 0$)?

```
def collatz(n):
    if n == 1:
        return 0
    elif n % 2 == 0:
        return 1 + collatz(n/2)
    else:
        return 1 + collatz(3*n+1)
```

- Who knows! It has been an open problem for years.

The collatz conjecture

By the XKCD webcomic



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

Repetition

- Suppose we want to make a function that given n calculates $\sum_{i=1}^{n} i$.

```
def sum_up_to(n):
    res = 1 + 2 + ... + n
    return res
```

This is not a valid program, for many reasons.

- Luckily, computers are very good at doing repetitive things. We have the `while` statement to aid us.

```
def sum_up_to(n):
    i = 1
    v = 0
    while i <= n:
        v = v + i
        i = i + 1
    return v
```

The body gets repeated while the condition evaluates to *true*.

Repetition

- Another handy construction is the `for` statement
- It goes through so called 'iterable' objects, e.g. strings

```
>>> for letter in 'hello':
...     print 'Give me an "' + letter + '"!'
...
Give me an "h"!
Give me an "e"!
Give me an "l"!
Give me an "l"!
Give me an "o"!
```

- 'Lists' are also iterable (we will see them later)

```
>>> range(3)
[0, 1, 2]
>>> for i in range(3):
...     print i**2
...
0
1
4
```

Repetition: while loops

- `while` loops are a powerful but tricky construction.
- They can run forever and make our program hang!

```
while True:
    x = x + 1
```

ok, we would not write that, but what about...

```
x = int(raw_input())
sum = 0
while x != 100:
    sum = sum + x
    x = x + 2
```

- If $x > 100$ or x is odd this loop never ends.

Repetition: loop invariants

- A *loop invariant* is an invariant used to prove properties of loops.
- For example, correctness and termination of loops.
- Connected to pre and post-conditions.

E.g.: `count(c:String, sentence:String)` $\rightarrow$ `res:Int`

- `pre:` `True`
- `post:` $res = |[1 : i \in \{0, \ldots, |sentence| - 1\}, \texttt{sentence}_i = \texttt{c}]|$

Suppose we have the following implementation

```
def count(c, sentence):
    i = 0; n = 0
    while i < len(sentence):
        if sentence[i] == c: n = n + 1
        i = i + 1
    return n
```

Repetition: loop invariants

$\texttt{post:}\ res = |[1 : i \in \{0, \ldots, |sentence| - 1\}, \texttt{sentence}_i = \texttt{c}]|$

```
def count(c, sentence):
    i = 0; n = 0
    while i < len(sentence):
        if sentence[i] == c: n = n + 1
        i = i + 1
    return n
```

Let **C** be our loop condition and **I** be our loop invariant, a theorem says:

$$\frac{\{C \wedge I\}\ \text{body}\ \{I\}}{\{I\}\ \textbf{while}\ (C)\ \text{body}\ \{\neg C \wedge I\}}$$

- **C:** $i < |\texttt{sentence}|$
- **I:** $0 \leq i \leq |\texttt{sentence}| \wedge n = |[1 : x \in \{0, \ldots, i - 1\}, \texttt{sentence}_x = \texttt{c}]|$

If we chose correctly our invariant, with $\neg C \wedge I$ we should be able to prove the postcondition.

Repetition: where's the catch?

Are the `for` and `while` statements equivalent?

- Short answer: in Python, yes.

Long answer:

- In old languages like BASIC and Pascal the `for` statement was meant to be used as `for i = A to B: body`. Modifications to `i` in the body would not change the iteration.
- In a `while` statement, the expression gets evaluated in every loop.

Some facts (check this out):

- In theoretical computer science the difference between `while` and `for` statements is kept.
- Using what we have seen you can write *any possible program*!
- But, if you don't use `while` you can only write 'some' of them.
- In fact, you could write any program using just ONE `while`.

The estimation game

- Building software in the real world is a lot about planning.
- Planning is a lot about dealing with uncertainty and deadlines.
- Your ability to do it right depends on: self-knowledge, experience in the field.

Take out a piece of paper, write your name and prepare yourself. You'll have to answer a set of questions with an interval (lower and upper bound)

- Average rainy days per year in Amsterdam $\rightarrow$ 188.
- Total area of Argentina (in km^2) $\rightarrow$ 2.780.400 km^2 (#8^{th}).
- Average pages of an ILLC MoL thesis $\rightarrow$ 77.

References

- Chapters 3 and 5–7 of the book
  http://greenteapress.com/thinkpython/thinkpython.html
- Wikipedia article on ‘Call Stack’
  http://en.wikipedia.org/wiki/Call_stack
- Wikipedia article on ‘Collatz conjecture’
  http://en.wikipedia.org/wiki/Collatz_conjecture
- Wikipedia article on ‘Loop invariants’
  http://en.wikipedia.org/wiki/Loop_invariant