

# Imperative programming with Python

## Class #3

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# Functions: execution and the call stack

```
► def cat_twice_and_print(part1, part2):  
    cat = part1 + part2  
    print_twice(cat)  
  
def print_twice(msg):  
    print msg  
    print msg  
  
line1 = 'welcome_  
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__main__  
line1 ↦ 'welcome '
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*cat\_twice\_and\_print*  
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*print\_twice*

*msg*  $\mapsto$  'welcome to the jungle'

*cat\_twice\_and\_print*

*part1*  $\mapsto$  'welcome '

*part2*  $\mapsto$  'to the jungle'

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 $\text{msg} \mapsto \text{'welcome to the jungle'}$

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line1 ↦ 'welcome '  
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Output:

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# Functions: execution and the call stack

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def cat_twice_and_print(part1, part2):  
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def print_twice(msg):  
    f(msg)  
    print msg  
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*print\_twice*  
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<i>__main__</i> $\text{line1} \mapsto \text{'welcome '}$ $\text{line2} \mapsto \text{'to the jungle'}$
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```
Traceback (most recent call last):  
  File "code.py", line 12, in <module>  
    cat_twice_and_print(line1, line2)  
  File "code.py", line 3, in cat_twice_and_print  
    print_twice(cat)  
  File "code.py", line 6, in print_twice  
    f(msg)  
NameError: global name 'f' is not defined
```

# Functions: recursion

- Functions can call *themselves* in their definition.

```
# calculates n * m (in a complicated way)
def multiply(n, m):
    if n == 0:
        return 0
    else:
        return m + multiply(n - 1, m)
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<i>multiply</i> $n \mapsto 2, m \mapsto 7$ $\text{ret} \mapsto 7 + \dots$
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<i>multiply</i> $n \mapsto 0, m \mapsto 7$ $\text{ret} \mapsto 0$
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<i>multiply</i> $n \mapsto 2, m \mapsto 7$ $\text{ret} \mapsto 7 + 7 = 14$
--

# Functions: recursion

- It is crucial that the arguments of a recursive call are in some sense 'smaller' than the arguments of the function call itself.
- What happens if we write `multiply` as follows

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def multiply(n, m):  
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```
>>> multiply(2, 7)  
Traceback (most recent call last):  
  File "<stdin>", line 1, in <module>  
  File "<stdin>", line 5, in multiply  
  File "<stdin>", line 5, in multiply  
  ...  
  File "<stdin>", line 5, in multiply  
RuntimeError: maximum recursion depth exceeded
```

- Stack overflow!

# Functions: recursion

- You can also have many recursive calls

```
def fib(n):  
    if n == 0:  
        return n  
    else:  
        return fib(n - 1) + fib(n - 2)
```

- Is it well defined?



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```
def fib(n):  
    if n == 0 or n == 1:  
        return n  
    else:  
        return fib(n - 1) + fib(n - 2)
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def fib(n):  
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```

- Is it well defined? Yes.

# Functions: recursion

- The argument itself *can* increase...

```
def reverse_string(s):  
    return reverse_from_n(s, 0)  
  
def reverse_from_n(s, i):  
    if i == len(s):  
        return ''  
    else:  
        return reverse_from_n(s, i+1) + s[i]
```

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```

- But if you look closer `len(s) - i` is strictly decreasing.

# Functions: recursion

- Is the following function well defined (for  $n > 0$ )?

```
def collatz(n):  
    if n == 1:  
        return 0  
    elif n % 2 == 0:  
        return 1 + collatz(n/2)  
    else:  
        return 1 + collatz(3*n+1)
```

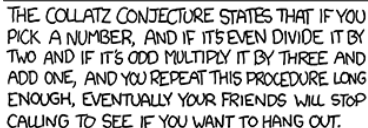
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- **Who knows!** It has been an open problem for years.

By the XKCD webcomic



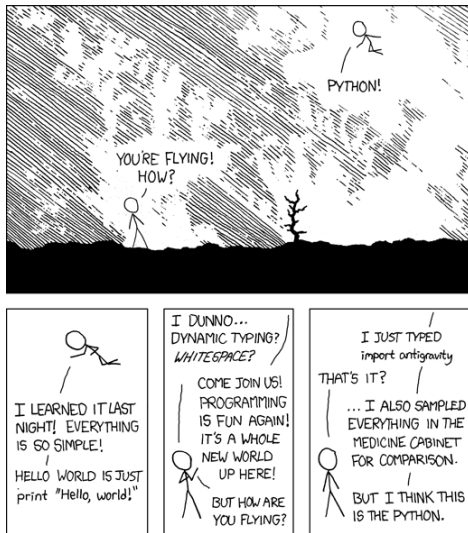


# Modules

- In general, programming languages come with a *library* of functions organized in some way.
- In Python, the library is organized in *modules*.
- For the moment, a module is a collection of related functions.
- Modules are used (imported) with the `import` keyword.

# Python module library

By the XKCD webcomic



# Using modules

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# Returns an integer from 1 to 10, endpoints included
>>> random.randint(1, 10)
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- **Suggested homework:** read the book's intro to the `math` module.

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>>> choice('abcdef')
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- But please **don't!** unless it is extremely necessary.

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- You can also import *everything* into the main namespace

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>>> from random import *
```

- But please **don't!** unless it is extremely necessary.
- You can import modules and assign them a different name

```
>>> import random as r
>>> r.randint(1, 10)
7
```

# Random numbers

By Dilbert

**DILBERT** By SCOTT ADAMS



- Related topic in Theoretical Computer Science:

[http://en.wikipedia.org/wiki/Algorithmically\\_random\\_sequence](http://en.wikipedia.org/wiki/Algorithmically_random_sequence)

# Repetition

- Suppose we want to make a function that given  $n$  calculates  $\sum_{i=1}^n i$ .

```
def sum_up_to(n):  
    res = 1 + 2 + ... + n  
    return res
```

This is not a valid program, for many reasons.

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    res = 1 + 2 + ... + n  
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```

This is not a valid program, for many reasons.

- Luckily, computers are very good at doing repetitive things. We have the `while` statement to aid us.

```
def sum_up_to(n):  
    i = 1  
    v = 0  
    while i <= n:  
        v = v + i  
        i = i + 1  
    return v
```

The body gets repeated while the condition evaluates to *true*.

# Repetition

- Another handy construction is the `for` statement
- It goes through so called 'iterable' objects, e.g. strings

```
>>> for letter in 'hello':  
...     print 'Give me an "' + letter + '!"'  
...  
Give me an "h!"  
Give me an "e!"  
Give me an "l!"  
Give me an "l!"  
Give me an "o!"
```

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```

- 'Lists' are also iterable (we will see them later)

```
>>> range(3)  
[0, 1, 2]  
>>> for i in range(3):  
...     print i**2  
...  
0  
1  
4
```

# Repetition: while loops

- `while` loops are a powerful but tricky construction.
- They can run forever and make our program hang!

```
while True:  
    x = x + 1
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```
while True:  
    x = x + 1
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ok, we would not write that, but what about...

```
x = int(raw_input())  
sum = 0  
while x != 100:  
    sum = sum + x  
    x = x + 2
```

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ok, we would not write that, but what about...

```
x = int(raw_input())  
sum = 0  
while x != 100:  
    sum = sum + x  
    x = x + 2
```

- If  $x > 100$  or  $x$  is odd this loop never ends.

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Suppose we have the following implementation

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def count(c, sentence):  
    i = 0; n = 0  
    while i < len(sentence):  
        if sentence[i] == c: n = n + 1  
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$$\frac{\{C \wedge I\} \text{ body } \{I\}}{\{I\} \text{ while } (C) \text{ body } \{\neg C \wedge I\}}$$

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If we chose correctly our invariant, with  $\neg C \wedge I$  we should be able to prove the postcondition.

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- Using what we have seen you can write *any possible program!*
- But, if you don't use `while` you can only write 'some' of them.
- In fact, you could write any program using just ONE `while`.

# References

- Chapters 3 and 5–7 of the book  
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